IV Semester B.A./B.Sc. Examination, May/June 2014
(NS) (Semester Scheme) (2012-2013 and Onwards)
MATHEMATICS (Paper – IV)

Time : 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) Define Ring and give an example.
2) In a Ring (R, +, .) prove that a. (b – c) = a.b – a.c and (b – a).a = b.a. – c.a $\forall$a, b, c $\in$ R.
3) Show that the subset $S = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \middle/ a, b \in z \right\}$ is a subring of ring $m_2(z)$.
4) Define right ideal of a ring.
5) It f : R $\to$ R' be a homorphism from the ring R into the ring R' then show that f(a – b) = f(a) – f(b) $\forall$ a, b $\in$ R.
6) Find the Fourier coefficient Q_0 for $f(x) = e^x$ in the interval $(-\pi, \pi)$.
7) Define half range Fourier cosine series for f(x) over $(0, \pi)$.
8) Obtain the Fourier series of the function f(x) = x over the interval $(-\pi, \pi)$.
9) Prove that the function $f(x, y) = \begin{cases} \dfrac{x^3 + y^3}{x^2 - y^2} & \text{for } (x, y) \neq (0,0) \\ 0 & (x, y) = (0,0) \end{cases}$ is continuous at (0, 0).
10) Expand the function $f(x, y) = e^{x+y}$ up to the second degree terms in powers of x and y.
11) Show that $f(x, y) = x^3 + y^3 - 3xy + 1$ is minimum at the point (1, 1).
12) Prove that p(n + 1) = n!.
13) Evaluate $\int_0^\infty e^{-x} x^{3/2} dx$.
14) Show that $\beta(m + 1, n) + \beta(m, n + 1) = \beta(m, n)$.

15) Solve $\frac{d^2y}{dx^2} - \frac{7dy}{dx} + 12y = 0$.

16) Evaluate $\frac{1}{D^2+9}\cos 3x$.

17) Find the part of complementary function of $xy'' - 2(x+1)y' + (x+2)y = (x-2)e^{2x}$.

18) Show that the equation $\sin x y'' - \cos x y' + 2y \sin x = 0$ is exact.

19) Evaluate $L\{e^{-t}\sin 2t\}$.

20) Find $L^{-1}\left\{\frac{S-1}{S^2+25}\right\}$.

II. Answer **any three** questions : **(3×5=15)**

1) Prove that every field is an integral domain.

2) A non empty subset S of a ring R to be a subring of R prove that

   i) $S + (-S) = S$ ii) $SS \leq S$

3) Let R be a ring of all 2 × 2 matrices with their elements as integers and $S = \left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \Big/ a, b \in z\right\}$. Show that S is a left ideal in R, but not a right ideal.

4) If $f : R \to R'$ be a homorphism of rings with kernel K, then f is one-one iff $K = \{0\}$.

5) If $R_1 = \left\{\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} a \in R\right\}$ where R is ring. Define $f : R_1 \to R$ by $f\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} = a$ for all $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \in R_1$, then show that f is an isomorphism.

III. Answer **any two** questions : **(2×5=10)**

1) Obtain the Fourier expansion of the function $f(x) = |x|$ in $(-\pi, \pi)$ and hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

2) If $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in $0 < x < 2\pi$ then show that $f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$.

3) Obtain the Fourier half range cosine series for the function $f(x) = \sin x$ in $(0, \pi)$.

IV. Answer **any two** questions : **(2×5=10)**

1) Expand $\cos(x + y)$ in powers of x and y upto third degree terms.

2) Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

3) Show that $\int_0^{\pi/2} \sin^p \theta . \cos^q \theta \, d\theta = \frac{1}{2} \beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$ where $p > -1$ $q > -1$ and hence evaluate $\int_0^{\pi/2} \sin^4 \theta . \cos^2 \theta \, d\theta$.

OR

Prove that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\tan\theta}} \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\tan\theta}} = \frac{\pi^2}{2}$.

4) Prove that $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = \beta(m,n)$ m, n > 0.

OR

Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

V. Answer **any four** questions : **(4×5=20)**

1) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \cos 3x$.

2) Solve $x\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \frac{1}{x}$.

3) Solve $\frac{dx}{dt} + 2y = -\sin t$. $\frac{dy}{dt} - 2x = \cos t$.

4) Solve $\sin^2x.\,y'' + \sin x.\cos x.\,y' + xy = 0$ by changing the independent variables.

5) Show that the equation $x^2(1+x)y'' + 2x(3x+2)y' + 2(3x+1)y = 0$ is exact and solve.

OR

Solve $y'' + 2y' + y = e^{-x}.\log x$ by the method of variation of parameter.

VI. Answer **any three** questions : **(3x5=15)**

1) Evaluate : i) $L\{\cos 5t.\cos 2t\}$ ii) $L\{(t+1)^2\}$

2) Evaluate $L^{-1}\left\{\frac{1}{S(S-1)(S-2)}\right\}$.

3) If $L\{f(t)\} = F(s)$ then prove that

1) $L\{f'(t)\} = SF(S) - f(0)$

2) $L\{(f''(t)\} = S^2F(S) - Sf(0) - f'(0)$

4) Solve using Laplace transform $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}$ $y(0) = 0,\ y'(0) = 0.$

5) Using convolution theorem find $L^{-1}\left\{\frac{1}{S(S+1)^2}\right\}$.
