III Semester B.A./B.Sc. Examination, Nov./Dec. 2007
(Semester Scheme)
MATHEMATICS (Paper – III)

Time : 3 Hours

Max. Marks : 90

Instructions : 1) *Answer* ***all*** *questions.*

2) *Answer should be written completely in* ***English*** *or in* ***Kannada****.*

I. Answer **any fifteen** of the following : **(15×2=30)**

1) Define order of an element of a group G. Find the order of an element of multiplicative group $\left\{1, w, w^2\right\}$.

2) Show that the group (z_5, t_5) is a cyclic group.

3) Find the number of generators of a cyclic group of order 60.

4) Let G be a cyclic group of order p and a be a generator. Prove that

$a^m = a^n \ (m \neq n)$ iff $m \equiv n \pmod{p}$.

5) Find all the cosets of $H = \{0, 3, 6, 9\}$ in (z_{12}, t_{12}).

6) Define index of a subgroup of a group.

7) Find the limit of a sequence

$$\left\{\frac{1^2+2^2+3^2+\ldots\ldots\ldots+n^2}{1+2+3+\ldots\ldots\ldots+n}\right\}$$

8) Find the nature of the sequence $\{1+\cos n\pi\}$

9) If $\lim\limits_{n\to\infty} x_n = l$, then prove that $\lim\limits_{n\to\infty} |x_n| = |l|$

10) If a series Σa_n is convergent then $\lim\limits_{n\to\infty} a_n = 0$.

11) State Cauchy's general principle of convergence.

12) Prove that $\sum \sin\left(\frac{1}{n}\right)$ is a divergent series.

13) Discuss the nature of the series $1^3 + 2^3 + 3^3 + \ldots + n^3 + \ldots$

14) State Raabe's test for infinite series.

15) Define limit of a function.

16) Verify the Roll's theorem for function $f(x) = x^2 - 6x + 8$ in the interval [2, 4].

17) Show that $a^x = 1 + \frac{x}{1!}(\log a) + \frac{x^2}{2!}(\log a)^2 + \frac{x^3}{3!}(\log a)^3 + \ldots$

18) Evaluate $\lim_{x \to 0} \log_x \tan x$.

19) Calculate a_0 in the Fourier series expansion of $f(x) = x + \frac{x^2}{4}$; $-\pi < x < \pi$

20) Obtain half range sine series for $f(x) = x$; $0 < x < 2$.

II. Answer **any three** of the following :

(3×5=15)

1) If a is any element of order of the group G prove that $a^m = e$ for any integer m if and only if n divides m.

2) If a is a generator of a cyclic group G, then show that O (a) = O (G).

3) If H is a subgroup of G, then show that there exists a one to one correspondence between any two right (left) cosets of H in G.

4) State and prove Lagrange's theorem for finite group.

5) Show that a group of prime order is a cyclic and also prove that it is an abelian.

III. Answer **any two** of the following : **(2×5=10)**

1) Prove that every convergent sequence is bounded.

2) Discuss the nature of the sequence $\left\{ n^{1/n} \right\}$. $\frac{n}{n} \lim\left(\frac{1}{n}\right)$

3) Examine the nature of the sequence

i) $n\left[\log(n+1) - \log n\right]$ $\frac{1}{n}$ $\frac{\lim \frac{1}{n}}{\left(\frac{1}{n}\right)}$

ii) $\left(1+\frac{a}{n}\right)^{n/b}$. $\frac{1}{n}$

IV. Answer **any three** of the following : **(3×5=15)**

1) State and prove D'Alembert's ratio test.

2) Examine the convergence of $1+\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+\ldots\ldots$ $1/x$

3) Discuss the convergence of $\sum \frac{\left[(n+1)x\right]^n}{n^{n+1}}$ tan log cot x

4) State and prove Leibnitz's rule for alternating series. x cosec x

5) Sum to infinity the series $\sum_{n=1}^{\infty}\left(\frac{n^2-2n-1}{n!}\right)$

V. Answer **any two** of the following : **(2×5=10)**

1) Prove that a function which is continuous in a closed interval attains its bounds.

2) State and prove Cauchy's mean value theorem.

3) Obtain by Maclaurin's theorem the first five terms in the expansion of $\log(1+\sin x)$.

4) Evaluate $\lim_{x \to 0}\left(\frac{1}{x}\right)^{\tan x}$. tan x log $\frac{1}{x}$ x tan x ~~sec~~ x log $1/x$

VI. Answer **any two** of the following : (2×5=

1) Find the Fourier expansion of $f(x) = x - x^2 ; -1 < x < 1$.

2) Obtain the Fourier series of $f(x) = x - 1$ in $(-\pi, \pi)$.

3) Find the half range cosine series for the function $f(x) = x$ in $(0, 2)$.

ಕನ್ನಡ ರೂಪಾಂತರ

ಸೂಚನೆಗಳು : 1) ಎಲ್ಲಾ ಪ್ರಶ್ನೆಗಳನ್ನು ಉತ್ತರಿಸಿ.
2) ಉತ್ತರಗಳನ್ನು ಸಂಪೂರ್ಣವಾಗಿ ಕನ್ನಡ ಅಥವಾ ಆಂಗ್ಲ ಭಾಷೆಯಲ್ಲಿ ಉತ್ತರಿಸಬೇಕು.

I. ಯಾವುದಾದರೂ ಹದಿನೈದು ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿ : (15×2=

1) ಸಂಕುಲದಲ್ಲಿ ಗಣಾಂಶಗಳ ಧಾತ್ವಾಂಶವನ್ನು ವ್ಯಾಖ್ಯಾನಿಸಿ. $\{1, w, w^2\}$ ಈ ಸಂಕುಲದ ಗಣಾಂಶಗಳ ಧಾತ್ವಾಂಶಗಳನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ.

2) (z_5, t_5) ಒಂದು ಚಕ್ರೀಯ ಸಂಕುಲ ಎಂದು ತೋರಿಸಿ.

3) 60 ಧಾತ್ವಾಂಶವುಳ್ಳ ಒಂದು ಚಕ್ರೀಯ ಸಂಕುಲದ ಉತ್ಪಾದಕ ಸಂಖ್ಯೆಗಳನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ.

4) G ಒಂದು p ಧಾತ್ವಾಂಶವುಳ್ಳ ಒಂದು ಚಕ್ರೀಯ ಸಂಕುಲನ. 'a' ಒಂದು ಉತ್ಪಾದಕ ಆದಾಗ $m \equiv n \pmod p$ ಆದರೆ ಮತ್ತು ಆದರೆ ಮಾತ್ರ $a^m = a^n \ (m \neq n)$ ಎಂದು ಸಾಧಿಸಿ.

5) (z_{12}, t_{12}) ಸಂಕುಲನದ ಉಪಸಂಕುಲನ $H = \{0, 3, 6, 9\}$ ಇದರ ಎಲ್ಲಾ ಕೊಸೆಟ್‌ಗಳನ್ನು ಕಂಡುಹಿಡಿಯಿರಿ.

6) ಒಂದು ಸಂಕುಲನದ ಉಪಸಂಕುಲನದ ಸೂಚಕವನ್ನು ವ್ಯಾಖ್ಯಾನಿಸಿ.

7) $\left\{ \dfrac{1^2 + 2^2 + 3^2 + \dots\dots + n^2}{1 + 2 + 3 + \dots\dots + n} \right\}$ ಈ ಅನುಕ್ರಮದ ಸೀಮಿತವನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ.

8) $\{1 + \cos n\pi\}$ ಈ ಅನುಕ್ರಮದ ಸ್ವಭಾವವನ್ನು ತಿಳಿಸಿ.

9) $\lim_{n \to \infty} x_n = l$ ಆದರೆ $\lim_{n \to \infty} |x_n| = |l|$ ಎಂದು ಸಾಧಿಸಿ.