III Semester B.A./B.Sc. Examination, Nov./Dec. 2010
(Semester Scheme)
MATHEMATICS (Paper – III)

Time : 3 Hours | Max. Marks : 90

***Instruction** : Answer **all** questions.*

I. Answer **any fifteen** of the following : **(15×2=30)**

1) In a finite group G, prove that $O(a) = O(a^{-1})$.

2) State the Lagrange's theorem for finite groups.

3) Show that group G = {2, 4, 6, 8} is cyclic under $\otimes_{10}$.

4) Find the index of the sub-group {0, 3} in the group $\{Z_6, \oplus_6\}$.

5) In a finite group G of order n, prove that $a^n = e, \forall a \in G$.

6) Find the order of all the elements of the group $(Z_4, \oplus_4)$.

7) Define a convergent sequence.

8) Find the limit of the sequence $\left\{\left(1+\frac{a}{n}\right)^{\frac{n}{b}}\right\}$.

9) Define a monotonic increasing sequence.

10) Show that the series $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ is convergent.

11) Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{n+2}{n^3+1}$.

12) Show that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\infty$ is convergent.

13) State D'Alembert's Ratio test for a series of positive terms.

14) Sum to infinity of $\frac{1}{2}+\frac{1}{3.2^2}+\frac{1}{5.2^3}+\ldots\infty$.

15) If f(x) is continuous at x = a, then prove that |f(x)| is also continuous at x = a.

16) Verify Rolle's theorem for the function $f(x) = x^2 - 6x+8$ in [2, 4].

17) Evaluate $\underset{x \to 0}{lt} \frac{\log \sin x}{\cot x}$.

18) Show that $\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$

19) Write the Fourier series expansion and expression for Fourier coefficients of a periodic function.

20) Find the half range sine series of f(x) = x in (0, π).

II. Answer **any three** of the following : **(3×5=15)**

1) Prove that evey sub-group of a cyclic group is cyclic.

2) Find all the generators of the cyclic group $(Z_{20}, \oplus_{20})$.

3) Find all the distinct cosets of the sub-group H = {1, 3, 9} of the group G = {1, 2, 3,...12} w.r.t.$\otimes_{13}$.

4) Prove that a group of prime order is cyclic and has no proper sub-groups.

5) If p is a prime number and 'a' is any integer then prove that $a^p \equiv a(\text{mod } p)$.

III. Answer **any two** of the following : **(2×5=10)**

1) Discuss the nature of the sequence $\left\{n^{\frac{1}{n}}\right\}$.

2) Prove that a monotonic decreasing sequence bounded below is convergent.

3) Examine the nature of the sequence

a) $\left\{\sqrt{n}(\sqrt{n+4}-\sqrt{n})\right\}$ b) $\left\{(-1)^n\left(\frac{n+1}{n}\right)\right\}$

IV. Answer **any three** of the following : **(3×5=15)**

1) State and prove Cauchy's root test for a series of positive terms.

2) Show that the series $\frac{1}{3}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+...$ is convergent.

3) Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{4.7.10...(3n+1)}{1.2.3...n}x^n$.

4) Examine the convergence of the series $1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+...\text{to }\infty$.

5) Find the sum of $\frac{1}{1.2.3}+\frac{1}{3.4.5}+\frac{1}{5.6.7}+...\text{to }\infty$

V. Answer **any two** of the following : **(2×5=10)**

1) Prove that a function defined and continuous on a closed interval is bounded.

2) State and prove Lagrange's mean value theorem.

3) Expand $\log_e(1+x)$ in Maclaurin's series upto the term containing x^4.

4) Evaluate : $\underset{x\to 0}{lt}(1+\sin x)^{\cot x}$.

VI. Answer **any two** of the following : **(2×5=10)**

1) Obtain the Fourier series of the periodic function of period 2π given by

$$f(x)=\begin{cases}-x, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$$ and hence deduced that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+...\frac{\pi^2}{8}$.

2) Obtain the Fourier series for the function $f(x) = x - x^2$ in $(-1, 1)$ of period 2.

3) Find the half range cosine series of the periodic function $f(x)=x^2$ in $(0, \pi)$.