MS – 278

II Semester B.A/B.Sc. Examination, May/June 2014
(Semester Scheme) (N.S.) (2011-12 and onwards)
MATHEMATICS – II

Time : 3 Hours Max. Marks : 100

Instruction : *Answer **all** the questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) Define order of an element of a group.

2) Prove that order of any power of an element of a group cannot exceed order of the element.

3) Find the number of generators of a cyclic group of integers under addition modulo12.

4) State Farmats theorem for finite group.

5) Find all the subgroups of $(Z_6, +_6)$.

6) Find the angle between radius vector and the tangent to the curve $r = a(1 - \cos\theta)$.

7) Find $\frac{ds}{dx}$ for the curve $ay^2 = x^3$.

8) Find the length of polar subtangent for the curve $r = a\cos 2\theta$ at $\theta = \frac{\pi}{6}$.

9) Find the Pedal equation of the curve $r = a(1 + \cos\theta)$.

10) Find the radius of curvature of the curve $9x^2 + 4y^2 = 36x$ at (2, 3).

11) Find the envelope of the family of circles.

$x^2 + y^2 - 2gx + g^2 - c^2 = 0$, where g being the parameter.

12) Find the asymptotes of the curve $r = \frac{a\theta}{\theta - 1}$.

13) Find the asymptotes parallel to the co-ordinate axes for the curve $(x^2 - a^2)\, y^2 = x^2\, (x^2 - 4a^2)$.

14) Find the area bounded by the curve $y = \sin^2 x$ and x – axis, between $x = 0$, $x = \frac{\pi}{2}$.

15) Write the formula for the volume obtained by revolving the arc of the curve $y = f(x)$ and the X-axis.

16) Solve $\frac{dy}{dx} = 1 + e^{x-y}$.

17) Solve $\frac{dy}{dx} - \frac{2y}{x} = x + x^2$.

18) Verify the exactness of the differential equation $(\cos x - 3x^2 \tan y)\, dx = x^3 \sec^2 y\, dy$.

19) Find the general and singular solution of $y = px + \log_e p$.

20) Find the orthogonal trajectories of the family of curves $ay^2 = x^3$.

II. Answer **any three** questions. **(3×5=15)**

1) If 'a' is any element of the group G, is of order 'n', then $a^m = e$ for any integer m if and only if n is a divisor of m.

2) If 'a' is a generator of a cyclic group G, then prove that 0 (a) = 0(G).

3) Show that a group of prime order is cyclic and also prove that it is abelian.

4) Define left and right cosets of a subgroup H of a group G. Find all the cosets of H = {0, 3, 6, 9} in a group of integer modulo 12 under addition.

5) State and prove Lagrange's theorem for a finite group.

III. Answer **any three** questions. (3×5=15)

1) With the usual notation prove that $\tan\phi = r\frac{d\theta}{dr}$.

2) Show that the curves $r = a\sec^2\frac{\theta}{2}$ and $r = b\,\text{cosec}^2\frac{\theta}{2}$ cut each other orthogonally.

3) Show that the Pedal equation of the curve $\frac{2a}{r} = 1-\cos\theta$ is $p^2 = ar$.

4) Find the radius of curvature for the curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$.

5) Find the circle of curvature for the curve $xy = c^2$ at (c, c).

IV. Answer **any two** questions. (2×5=10)

1) Show that the curve $r = \frac{a\theta^2}{\theta^2 - 1}$ has a point of inflexion at $r = \frac{3a}{2}$.

2) Find all the asymptotes of the curve $x^3 + y^3 - 3axy = 0$.

3) Find the position and nature of the double points of the curve $a^4y^2 = x^4(2x^2 - 3a^2)$.

4) Trace the curve catenary $y = c\cosh\left(\frac{x}{c}\right)$, $c > 0$.

V. Answer **any two** questions. (2×5=10)

1) Find the length of arc of cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.

2) Find the surface area of the hemisphere of radius ä .

3) Find the volume of the solid generated by revolving the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, $a > 0$ about x – axis.

VI. Answer **any four** questions : **(4×5=20)**

1) Solve $(x^2 + y^2)\, dy = xy dx$.

2) Solve $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$.

3) Solve $x\frac{dy}{dx} + y = y^2 \log x$.

4) Verify for exactness and solve.
$[\cos x \tan y + \cos(x + y)]\, dx + [\sin x \sec^2 y + \cos(x + y)]\, dy = 0$.

5) Find the general and singular solution of $y = px + \frac{a}{p}$.

6) Show that the family of curves.
$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$, where λ is a parameter is self orthogonal.

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