



**II Semester B.A/B.Sc. Examination, May/June 2014  
(Semester Scheme) (N.S.) (2011-12 and onwards)  
MATHEMATICS – II**

Time : 3 Hours

Max. Marks : 100

***Instruction : Answer all the questions.***

I. Answer any fifteen questions :

(15×2=30)

- 1) Define order of an element of a group.
- 2) Prove that order of any power of an element of a group cannot exceed order of the element.
- 3) Find the number of generators of a cyclic group of integers under addition modulo 12.
- 4) State Fermat's theorem for finite group.
- 5) Find all the subgroups of  $(\mathbb{Z}_6, +_6)$ .
- 6) Find the angle between radius vector and the tangent to the curve  $r = a(1 - \cos \theta)$ .
- 7) Find  $\frac{ds}{dx}$  for the curve  $ay^2 = x^3$ .
- 8) Find the length of polar subtangent for the curve  $r = a \cos 2\theta$  at  $\theta = \frac{\pi}{6}$ .
- 9) Find the Pedal equation of the curve  $r = a(1 + \cos \theta)$ .
- 10) Find the radius of curvature of the curve  $9x^2 + 4y^2 = 36x$  at (2, 3).
- 11) Find the envelope of the family of circles.  
 $x^2 + y^2 - 2gx + g^2 - c^2 = 0$ , where  $g$  being the parameter.
- 12) Find the asymptotes of the curve  $r = \frac{a\theta}{\theta - 1}$ .

P.T.O.

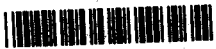


- 13) Find the asymptotes parallel to the co-ordinate axes for the curve  
 $(x^2 - a^2) y^2 = x^2 (x^2 - 4a^2)$ .
- 14) Find the area bounded by the curve  $y = \sin^2 x$  and  $x$  - axis, between  $x = 0$ ,  
 $x = \frac{\pi}{2}$ .
- 15) Write the formula for the volume obtained by revolving the arc of the curve  
 $y = f(x)$  and the X-axis.
- 16) Solve  $\frac{dy}{dx} = 1 + e^{x-y}$ .
- 17) Solve  $\frac{dy}{dx} - \frac{2y}{x} = x + x^2$ .
- 18) Verify the exactness of the differential equation  $(\cos x - 3x^2 \tan y) dx = x^3 \sec^2 y dy$ .
- 19) Find the general and singular solution of  $y = px + \log_e p$ .
- 20) Find the orthogonal trajectories of the family of curves  $ay^2 = x^3$ .

II. Answer any three questions.

(3×5=15)

- 1) If 'a' is any element of the group G, is of order 'n', then  $a^m = e$  for any integer m if and only if n is a divisor of m.
- 2) If 'a' is a generator of a cyclic group G, then prove that  $O(a) = O(G)$ .
- 3) Show that a group of prime order is cyclic and also prove that it is abelian.
- 4) Define left and right cosets of a subgroup H of a group G. Find all the cosets of  $H = \{0, 3, 6, 9\}$  in a group of integer modulo 12 under addition.
- 5) State and prove Lagrange's theorem for a finite group.



III. Answer any three questions.

(3×5=15)

- 1) With the usual notation prove that  $\tan \phi = r \frac{d\theta}{dr}$ .
- 2) Show that the curves  $r = a \sec^2 \frac{\theta}{2}$  and  $r = b \operatorname{cosec}^2 \frac{\theta}{2}$  cut each other orthogonally.
- 3) Show that the Pedal equation of the curve  $\frac{2a}{r} = 1 - \cos \theta$  is  $p^2 = ar$ .
- 4) Find the radius of curvature for the curve  $x = a(t - \sin t)$ ,  $y = a(1 - \cos t)$ .
- 5) Find the circle of curvature for the curve  $xy = c^2$  at  $(c, c)$ .

IV. Answer any two questions.

(2×5=10)

- 1) Show that the curve  $r = \frac{a\theta^2}{\theta^2 - 1}$  has a point of inflexion at  $r = \frac{3a}{2}$ .
- 2) Find all the asymptotes of the curve  $x^3 + y^3 - 3axy = 0$ .
- 3) Find the position and nature of the double points of the curve  $a^4 y^2 = x^4(2x^2 - 3a^2)$ .
- 4) Trace the curve catenary  $y = c \cosh \left( \frac{x}{c} \right)$ ,  $c > 0$ .

V. Answer any two questions.

(2×5=10)

- 1) Find the length of arc of cycloid  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ .
- 2) Find the surface area of the hemisphere of radius  $a$ .
- 3) Find the volume of the solid generated by revolving the astroid  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ ,  $a > 0$  about  $x$  - axis.



VI. Answer **any four** questions :

(4×5=20)

1) Solve  $(x^2 + y^2) dy = xy dx$ .

2) Solve  $(1 + x^2) \frac{dy}{dx} + y = e^{\tan^{-1}x}$ .

3) Solve  $x \frac{dy}{dx} + y = y^2 \log x$ .

4) Verify for exactness and solve.

$$[\cos x \tan y + \cos(x + y)] dx + [\sin x \sec^2 y + \cos(x + y)] dy = 0.$$

5) Find the general and singular solution of  $y = px + \frac{a}{p}$ .

6) Show that the family of curves.

$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1, \text{ where } \lambda \text{ is a parameter is self orthogonal.}$$

---