

Second Semester B.Sc Examination, May / June 2006
(Semester Scheme)

MATHEMATICS (Paper – II)

Time : 3 Hours

Max. Marks : 90

Instructions : 1) Answer **all** questions.

2) Answers should be written completely **either in English or in Kannada** .

I. Answer any Fifteen of the following :

15 x 2 = 30

- 1) Reduce the matrix $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$ to echelon form
- 2) Find the value of 'a' if the matrix $\begin{bmatrix} 2 & 4 & -4 & a \\ -1 & -2 & -1 & 2 \\ 1 & 2 & -1 & 3 \end{bmatrix}$ is of rank 2.
- 3) Find the value of λ for which the following system $2x - y + 2z = 0$,
 $3x + y - z = 0$, $\lambda x - 2y + z = 0$ has a non trivial solution.
- 4) If λ is an eigen value of a square matrix A, prove that λ^2 is an eigen value of A^2
- 5) Show that a square matrix A and its transpose A' have the same eigen values.
- 6) Find the angle between the radius vector and the tangent for the curve
 $r = a(1 + \sin \theta)$ at $\theta = \frac{\pi}{6}$
- 7) Show that for the curve $r = a\theta$ the polar subtangent varies as the square of the radius vector.
- 8) With usual notations prove that $p = r \sin \phi$ for a polar curve $r = f(\theta)$
- 9) Find $\frac{ds}{dx}$ for the curve $y = a \log \sec \left(\frac{x}{a} \right)$
- 10) Define Evolute and involute of a curve.
- 11) Show that the curve $y = x^3$ has a point of inflexion at the origin.
- 12) Find the envelope of the family of curves $(x - a)^2 + y^2 = 4a$, where 'a' is the parameter
- 13) Find the asymptotes parallel to the coordinate axis of the curve $x^2y^2 - a^2x^2 = a^2y^2$
- 14) Find the area of the lemniscate $r^2 = a^2 \cos 2\theta$

15) Find the volume generated by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ rotated about the y-axis

16) Solve $\frac{dy}{dx} + 1 = e^{(x+y)}$

17) Solve $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$

18) Verify the equation $(4x+3y+1) dx + (3x+2y+1) dy = 0$ for exactness and hence solve it.

19) Find the general and singular solutions of $y = px + \log p$

20) Find the orthogonal trajectories of the curve $r = a\theta$

II. Answer any three of the following :

3 x 5 = 15

1) Find the inverse of the matrix A by elementary transformations where

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

2) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 4 & 1 & -1 \\ -3 & 2 & 3 & 1 \\ 0 & 7 & 6 & 3 \end{bmatrix}$

3) Show that the equation $x + y + z = 6$, $x + 2y + 3z = 14$, $x + 4y + 7z = 30$ are consistent and hence solve.

4) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

5) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

III. Answer any two questions :

2 x 5 = 10

1) With the usual notations show that $\tan \phi = r \cdot \frac{d\theta}{dr}$ for a polar curve $r = f(\theta)$

2) Find the pedal equation of the curve $y^2 = 4a(x+a)$

3) Find the radius of curvature at any point 't' on the Astroid $x = a \cos^3 t$; $y = a \sin^3 t$

4) Find the coordinates of the centre of curvature of the cycloid

$x = a(t - \sin t)$ $y = a(1 - \cos t)$ and show that its evolute is another cycloid.

IV. Answer any two questions.

2 x 5 = 10

- 1) Find the range of values of x for which $y = 3x^5 - 40x^3 + 3x - 20$ is (i) concave (ii) convex. Also find the points of inflexion.
- 2) Find all the asymptotes of the curve $xy^2 + x^2y + xy + y^2 + 3x = 0$
- 3) Find the position and nature of the double points on the curve $x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$
- 4) Trace the curve (cis soid) $y^2(a - x) = x^3$, $a > 0$

V. Answer any two questions :

2 x 5 = 10

- 1) Find the surface area of the sphere of radius a if the surface is generated by a semicircle revolving about the x axis.
- 2) Find the area of the Astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$
- 3) Find the volume of the solid obtained by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line.

VI. Answer any three questions.

3 x 5 = 15

- 1) Solve : $\frac{dy}{dx} + \frac{x+y \cos x}{1+\sin x} = 0$
- 2) Solve : $y - x \frac{dy}{dx} = a \left(y^2 + \frac{dy}{dx} \right)$
- 3) Find the general and singular solutions of $x^2(y - px) = yp^2$ by using the substitution $x^2 = u$, $y^2 = v$.
- 4) Show that the family of curves $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$ (λ is the parameter) is self orthogonal.
